WHITE HOLE, BLACK WHOLE, AND THE BOOK

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Dedicated to the memory of Professor Paul Erdös, the originator of The Book.

ABSTRACT. Physical and intellectual spaces are visualized making use of concepts from intuitive set theory. Intellectual space is defined as the set of all proofs of mathematical logic, contained in The Book conceived by Erdös.

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1. INTRODUCTION

In an earlier paper [1], it was shown that Zermelo-Fraenkel set theory gets considerably simplified, if we add two axioms, *Monotonicity* and *Fusion*, to it. In the resulting intuitive set theory (IST), the continuum hypothesis is a theorem, axiom of choice is a theorem, Skolem Paradox does not crop up, non-Lebesgue measurable sets are not possible, and the unit interval splits into a set of infinitesimals with cardinality $\aleph_0$ [1, 2]. This paper shows that IST can be used to visualize the infinite physical space around us as a set. Further, if we consider all the proofs of mathematics as our intellectual space, then IST provides a way to consider that also as a set.

2. WHITE HOLE

The axiom of fusion allows us to imagine a unit interval as a set of infinitesimals, with each infinitesimal containing $\aleph_1$ *figments* (elements which cannot be accessed by the axiom of choice) in it. We consider these infinitesimals as integral units which cannot be broken up any further. To facilitate the discussion, in addition to Dedekind cuts, we will use also the concept of a *Dedekind knife*, and assume that the knife can cut any interval given to it, *exactly* in the middle. From this, it follows that every infinite *recursive* subset of positive integers, or equivalently, a binary number in the unit interval, represents the use of Dedekind knife an infinite number of times. The result we get when we use the knife $\aleph_0$ times, according to an infinite binary sequence, is what we call an infinitesimal and the location of that infinitesimal is what we call a number.

What if, the operation of the knife is continued further an infinite number of times according to a new arbitrary infinite binary sequence. We can see the intuitionists protesting at this stage, that you cannot start another infinite sequence before, you have *completed* the previous infinite sequence. For this, the formalist answer is that, in mathematics, there is no harm in imagining things which cannot be accomplished physically. We can see here, the source of the oxymoron *completed infinity*, and the motivation for our definition of a *bonded set*. Bonded set is a set, from which axiom of choice cannot pick an element and separate it. These special elements, we call figments.

Having stated this, we continue our second infinite cutting, this time, without bothering to restrict ourselves to recursive subsets of positive integers as in the original case. The justification for this is that our operation is in the realm of the imaginable and not physical. The result of the cutting is $\aleph_1$ figments and they are to be considered only as a figment of our imagination. These arguments, of course, do not prove the consistency of the axiom of fusion, but hopefully makes it plausible.

The discussion above allows us to define a *white hole* (WhiteHole, white-hole) as the infinitesimal (bonded set) corresponding to an infinite recursive subset of positive integers (a binary number in the unit interval). It represents an indefinitely small void, which cannot be broken up any further. However, it does contain $\aleph_1$ figments which cannot be isolated.

3. BLACK WHOLE

A binary number is usually defined as a two way infinite binary sequence around the binary point,

$$\dots 000xx\dots xxx.xxxxx\dots$$

in which the xs represent either a 0 or 1, and the infinite sequence on the left eventually ends up in 0s. The two's complement number system represents a negative number by a two way infinite sequence,

$$\ldots 111xx\ldots xxx.xxxxx\ldots$$

in which the infinite sequence on the left eventually ends up in 1s.

We define the *Universal Number System* as the number system in which there are no restrictions on the infinite sequences on both sides.

It is easy to recognize that, the sequence

$$\ldots 00000.xxxxx\ldots$$

with a nonterminating binary sequence on the right side represents a number in a unit interval and also a whitehole. The concept of a *black whole* (Black-Whole, blackwhole) is now easy to define. The two way infinite sequence we get when we flip the whitehole around the binary point,

$$\ldots xxxxx.00000\ldots$$

represents a *supernatural number* and also a *black stretch*. The infinite set of black stretches, we define as the blackwhole. Thus, blackwhole can be considered as a dual of the unit interval. The name black stretch is supposed to suggest that it can be visualized as a set of points distributed over an infinite line, but it should be recognized that it is a bonded set, which the axiom of choice cannot access. Our description of the black whole clearly indicates that it can be used to visualize what is beyond the finite physical space around us.

4. THE BOOK

The Book (TheBook, the book) as originally conceived by Erdös is a book that contains all the smallest proofs of mathematics arranged in the lexical order. Since the alphabet of any axiomatic theory is finite, and every proof is a well-defined formula, it follows that a computer can be set up to start writing this book. We cannot expect the computer to stop, since there are an infinite number of proofs in mathematics. Thus, a computer generated book will always have to be unfinished, the big difference between a computer generated book and The Book is that it is a *finished* book.

The physical appearance of the book can be visualized as below.

- The front cover and the back cover are each one millimeter thick, and the entire book, including the covers, is three millimeters thick.

- The first sheet of paper is half-millimeter thick, the second sheet is half thick as the first, the third sheet is half thick as the second, and so on.
- On every odd page is written a full proof, and in the next even page is written the corresponding theorem.
- The last sheet is stuck with the cover with the result that the Last Theorem is not visible.

From the description of the book, we can infer that any formula which is a theorem can be found in the book, by sequentially going through the pages of the book. The only difficulty is that, if a formula is not a theorem, we will be eternally searching for it.

5. CONCLUSION

The upshot of all our discussion, reduces to the following. We live in a space where the visible finite part is filled up with white holes. When we use Dedekind knife an infinite number of times to cut a line segment we get a real number and a white hole. The invisible part of the physical space is an unimaginably large, unreachable black whole, comprising of transfinite black stretches. If we use the concept of point at infinity of complex analysis, the black whole can be visualized as the usual black hole. The Book allows us to read through proofs and collect as many theorems as we want, but it does not help us very much in deciding whether a given formula is a theorem. The main problem of mathematics is to write a New Book with the *theorems* listed in lexical order. Hilbert once had hopes of setting up a computer to start writing this book, but a great achievement of the twentieth century is the dashing of that hope.

1. K. K. Nambiar, *Intuitive Set Theory*, Computers and Mathematics with Applications **39** (2000), no. 1-2, 183–185.
2. ______, *Visualization of Intuitive Set Theory*, Computers and Mathematics with Applications **41** (2001), no. 5-6, 619–626.

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