Generalized Continuum Hypothesis and the Axiom of Combinatorial Sets

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Prologue

In an earlier paper [1], intuitive set theory (IST) was defined as the theory we get when we add the two axioms, *monotonicity* and *fusion*, to ZF theory. Here we attempt to replace the axiom of monotonicity with a simpler axiom we call, *axiom of combinatorial sets*.

Axiom of Combinatorial Sets

If k is an ordinal, we write $\binom{\aleph_\alpha}{k}$ for the cardinality of the set of all subsets of $\aleph_\alpha$ with cardinality of k.

Axiom of Combinatorial Sets:

$$\aleph_{\alpha+1} = \binom{\aleph_\alpha}{\aleph_\alpha}.$$

Derivation

We derive the Generalized Continuum Hypothesis from the axiom of combinatorial sets as below:

$$2^{\aleph_\alpha} = \binom{\aleph_\alpha}{0} + \binom{\aleph_\alpha}{1} + \binom{\aleph_\alpha}{2} + \cdots \binom{\aleph_\alpha}{\aleph_0} + \cdots \binom{\aleph_\alpha}{\aleph_\alpha}.$$

Note that $\binom{\aleph_\alpha}{1} = \aleph_\alpha$. Since, there are $\aleph_\alpha$ terms in this addition and $\binom{\aleph_\alpha}{k}$ is a monotonically nondecreasing function of k, we can conclude that

$$2^{\aleph_\alpha} = \binom{\aleph_\alpha}{\aleph_\alpha}.$$

Using axiom of combinatorial sets, we get

$$2^{\aleph_\alpha} = \aleph_{\alpha+1}.$$

Epilogue

In view of the fact that we can derive the generalized continuum hypothesis from the axiom of combinatorial sets, we can replace the axiom of monotonicity [1, 2] with the axiom of combinatorial sets, in the definition of intutive set theory.

References

[1] K. K. Nambiar, *Intuitive Set Theory*, Computers and Mathematics with Applications **39** (1999), no. 1-2, 183–185.

[2] K. K. Nambiar, *Real Set Theory*, Computers and Mathematics with Applications **38** (1999), no. 7-8, 167–171.